On the Complete Pivoting Conjecture for Hadamard Matrices of Small Orders

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In this paper we study explicitly the pivot structure of Hadamard matrices of small orders 16,20 and 32. An algorithm computing the (n–j) × (n–j) minors of Hadamard matrices is presented and its implementation for n = 12 is described. Analytical tables summarising the pivot patterns attained are given.

Keywords: Gaussian elimination, pivot size, complete pivoting, minors, Hadamard matrices.

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1. INTRODUCTION

Let A be an $n \times n$ real matrix, and let $\underline{b}$ be a real n-vector. In his fundamental work on backward error analysis Wilkinson(1963) proved that when the linear system $A \cdot \underline{x} = \underline{b}$ is solved in floating point arithmetic by Gaussian elimination (GE) with either partial or complete pivoting the computed solution $\hat{\underline{x}}$ satisfies

$$(A+E) \cdot \hat{\underline{x}} = \underline{b}$$

where the norm of the perturbation matrix E can be bounded from above as follows:

$$\|E\|_\infty \leq g(n,A) \cdot f(n) \cdot u \, \|A\|_\infty \qquad (1)$$

where u is the unit roundoff, $f(n)$ is a cubic polynomial of n, and g(n,A) is the growth factor defined by

$$g(n, A) = \frac{\max_{i,j,k} |a_{ij}^{(k)}|}{|a_{11}^{(0)}|}$$

where $a_{ij}^{(k)}$, $k = 1,2,..., n–1$ denotes the (i,j) th element that occurs at the k-th step of elimination. The elements $a_{ii}^{(n-1)}$ are called pivots.

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